Version : 1.0 0609

General Certificate of Education

Mathematics 6360

MPC2 Pure Core 2

Mark Scheme

2009 examination - June series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Set and published by the Assessment and Qualifications Alliance.

Key to mark scheme and abbreviations used in marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
$-x$ EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MPC2

Q	Solution	Marks	Total	Comments
1(a)	$5^2 = 7^2 + 8^2 - 2 \times 7 \times 8\cos\theta$	M1		Use of the cosine rule – must be correct (PI by the correct line below)
	$\cos\theta = \dfrac{7^2+8^2-5^2}{2\times7\times8}$ $\left(= \dfrac{88}{112} = 0.7857\ldots\right)$	m1		Rearrangement
	$\theta = 38.21\ldots. = 38.2°$ (to nearest 0.1°)	A1	3	CSO (Must see either exact value for $\cos\theta$ or at least 4sf value for either $\cos\theta$ or θ before the printed answer 38.2°) AG
(b)	Area = $\dfrac{1}{2}\times7\times8\sin\theta$	M1		OE eg Area = $\sqrt{10(10-5)(10-8)(10-7)}$ $(= \sqrt{300}\,)$
	= 17.3 {cm^2} to 3sf	A1	2	Condone 17.31 to 17.33 inclusive
	Total		**5**	
2(a)	$(n =) -4$	B1	1	Accept x^{-4}
(b)	$\left(1+\dfrac{3}{x^2}\right)^2 = 1+\dfrac{6}{x^2}+\dfrac{9}{x^4}$	B2,1,0	2	Apply ISW after B2 stage (B1 if correct but unsimplified seen)
(c)	$\int \left(1+\dfrac{3}{x^2}\right)^2 \mathrm{d}x = x - 6x^{-1} - 3x^{-3} + c$	M1		At least one power of x correctly obtained in the integration of an expansion
		A2,1,0	3	A2 terms correct **and** '$+ c$' (A1F two terms in x correct ft on expansion provided integrating x to a negative power)
(d)	$\int_1^3 \left(1+\dfrac{3}{x^2}\right)^2 \mathrm{d}x = \left[x-\dfrac{6}{x}-\dfrac{3}{x^3}\right]_1^3$			
	$= \left(3-\dfrac{6}{3}-\dfrac{3}{27}\right)-(1-6-3)$	M1		Dealing correctly with limits; F(3) − F(1) (must have attempted integration to get F)
	$= 8\dfrac{8}{9}$	A1	2	CSO; OE provided value is **exact**, eg $\dfrac{80}{9}, \dfrac{240}{27}$; ISW dec value after exact value seen NMS scores 0/2
	Total		**8**	

MPC2 (cont)

Q	Solution	Marks	Total	Comments
3(a)	$24 = 16k + 12$ $k = 12 \div 16 = 0.75$	M1 A1	 2	Condone with 0.75 (OE) subst for k AG; OE fraction; if verification must explicitly state the conclusion
(b)	$u_3 = 30$ $u_4 = 34.5$	B1 B1F	 2	 ft on 0.75 × cand's u_3 + 12
(c)(i)	$L = 0.75L + 12$	M1	1	Replacing u_{n+1} and u_n by L
(ii)	$L = \dfrac{12}{1-k} = \dfrac{12}{1-0.75}$ $L = 48$	m1 A1	 2	PI, but previous M **must** be scored SC: (c)(i) incorrect and then in (c)(ii) $L = 0.75L + 12$ leading to $L = 48$ scores B2
	Total		**7**	
4(a)	$h = 2$ $g(x) = \sqrt{x^3+1}$ $I \approx h/2\{\ldots\}$ $\{\ldots\} = g(0) + g(6) + 2[g(2) + g(4)]$	B1 M1		PI OE summing of areas of the 'trapezia'.. Can award even if MR expression for g(x) but must be using from 0 to 6
	$\{\ldots\} = 1 + \sqrt{217} + 2(3 + \sqrt{65})$ $1 + 14.73... + 2(3 + 8.06\ldots)$	A1		OE Accept 2dp evidence for surds
	$(I \approx)$ 37.8554… = 37.86 (to 4sf)	A1	4	Must be 37.86
(b)	$f(x) = \sqrt{(2x)^3+1} = \sqrt{8x^3+1}$	M1 A1	 2	$\sqrt{kx^3+1}$, $k \neq 1$ or 0 or $f(x) = g(2x)$ Either form acceptable
	Total		**6**	

MPC2 (cont)

Q	Solution	Marks	Total	Comments
5(a)	$\frac{dy}{dx} = \frac{45}{2}x^{\frac{1}{2}} - \frac{5}{2}x^{\frac{3}{2}}$	M1 A2,1,0	3	One power correctly obtained A1 for each term on the RHS coeffs simplified
(b)	$\frac{45}{2}x^{\frac{1}{2}} - \frac{5}{2}x^{\frac{3}{2}} = 0$	M1		cand's (a) = 0
	$\frac{5}{2}x^{\frac{1}{2}}(9 - x) = 0$	m1		Must be solving eqn of form $ax^m + bx^n = 0$, m and n non-zero, with at least one of m and n non-integer and reaching a stage from which the non-zero value of x can be stated PI. Must deal with powers of x correctly and any squaring of kx^p terms or expressions must be correct.
	At M, $x = 9$	A1		
	$y_M = 162$	A1	4	M1 must be scored, else 0/4
(c)	At $P(1, 14)$, $\frac{dy}{dx} = \frac{45}{2} - \frac{5}{2} = 20$	M1		Attempt to find $y'(1)$
	Tangent at P: $y - 14 = m(x - 1)$	m1		m = cand's value of $y'(1)$
	$y - 14 = 20x - 20$; $y = 20x - 6$	A1	3	CSO; AG
(d)	Tangent at M: $y = 162$	B1F		ft y = cand's y_M
	At R, $162 = 20x - 6$; $x = 8.4$	M1		Solving cand's numerical $y_M = 20x - 6$ to find a value for x
	Distance $RM = \lvert x_M - x_R \rvert = 9 - 8.4 = 0.6$	A1F	3	ft on coordinates of M
	Total		**13**	
6	{Area of sector =} $\frac{1}{2}r^2\theta$	M1		$\frac{1}{2}r^2\theta$ seen or used for the area; PI
	$r^2 = \frac{33.75}{\frac{1}{2}\theta}$ (= 56.25)	m1		Correct rearrangement to $r^2 = \ldots$ or $r = \ldots$
	$r = 7.5$ {Arc =} $r\theta$ …. = 9	A1 M1 A1F		PI eg by a correct arc length $r\theta$ seen or used for the arc length ft on 1.2 × cand's r provided the **two** M's scored; if not explicit, PI by ft on 3.2 × cand's r for perimeter
	{Perimeter =} 24 {cm}	A1	6	CAO
	Total		**6**	

MPC2 (cont)

Q	Solution	Marks	Total	Comments
7(a)(i)	$ar = 375$; $ar^4 = 81$	B1		For either OE or PI by next line
	$\Rightarrow 375r^3 = 81$	M1		Elimination of a OE
	$r^3 = \dfrac{81}{375} = \dfrac{27}{125} = 0.216 \Rightarrow r = 0.6$	A1	3	CSO AG Full valid completion SC: Clear explicit verification, with statement max B1 out of 3. (If considers uniqueness then 3 is possible)
(ii)	$0.6a = 375$	M1		OE; PI
	$a = 625$	A1	2	
(b)	$\dfrac{a}{1-r} = \dfrac{a}{1-0.6}$	M1		$\dfrac{a}{1-r}$ **used** with $\lvert$ value of $r \rvert < 1$
	$S_\infty = \dfrac{625}{0.4} = 1562.5$	A1F	2	ft on cand's value for a…. ie $2.5 \times a$
(c)	$\sum_{n=6}^{\infty} u_n = \sum_{n=1}^{\infty} u_n - \sum_{n=1}^{5} u_n$	M1		
	$u_3 = 0.6u_2$ (= 225) and $u_4 = 0.6^2 u_2$ (= 135)	M1		Valid method to either find u_3 and u_4 or use of $\{S_n =\}\dfrac{a(1-r^n)}{1-r}$ for either $n = 5$ or $n = 6$
	$\sum_{n=1}^{5} u_n = 625+375+225+135+81$ (= 1441)	A1		
	$\sum_{n=6}^{\infty} u_n = 1562.5 - 1441 = 121.5$	A1	4	
	Alternative for (c):			
	Recognise that the sum to infinity with first term u_6 is required	(M1)		
	Valid method to find u_6 $(= 0.6u_5)$	(M1)		
	$\sum_{n=6}^{\infty} u_n = \dfrac{81 \times 0.6}{1-0.6}$	(A1)		
	$= 121.5$	(A1)		
	Total		**11**	

MPC2 (cont)

Q	Solution	Marks	Total	Comments
8(a)	$\dfrac{\sin\theta}{\cos\theta}-\dfrac{\cos\theta}{\cos\theta}=4$			
	$\tan\theta-1=4$	M1		$\tan\theta=\dfrac{\sin\theta}{\cos\theta}$ stated or used
	$\tan\theta=5$	A1	2	AG; CSO
(b)(i)	$2\cos^2 x-\sin x=1$ $2(1-\sin^2 x)-\sin x=1$	M1		**Use** of $\cos^2 x+\sin^2 x=1$
	$2-2\sin^2 x-\sin x=1$ $\Rightarrow\ 2\sin^2 x+\sin x-1=0$	A1	2	AG; CSO
(ii)	$(\sin x+1)(2\sin x-1)\ =0$	M1		Factorisation or use of formula; PI by **both** correct values for $\sin x$
	$\sin x=-1,\quad \sin x=0.5$	A1		Need both
	$(\sin x=-1)$ so $x=270^\circ$	B1		
	$(\sin x=0.5)$ so $x=30^\circ$	A1		30° as the only acute angle
	$x=180-30=150^\circ$	B1F	5	ft for 2nd angle from c's $\sin x$ = non-integer Ignore values outside interval 0º–360º but extras inside interval lose the corresp. B, A or B1F mark. If using rads, accepting either equivalent exact vals (in terms of pi) or 2dp values instead of degrees, penalise max of 1 mark from any of the final three marks (B1A1B1F) awarded NMS: 270º (B1); 30º, 150º (B1) [max 2/5]
	Total		**9**	

MPC2 (cont)

Q	Solution	Marks	Total	Comments
9(a)(i)	$\sqrt{125} = \sqrt{25\times5} = 5\sqrt{5}$	M1		OE eg $\sqrt{125} = \sqrt{5^3}$ or $5^{1.5}$ seen
	$5^p = \sqrt{125} \Rightarrow p = 1.5$	A1	2	Correct value of p must be explicitly stated
	Alternative for (a)(i):			
	$p\log5 = \frac{1}{2}\log125$	(M1)		OE eg $p\log5 = \log11.18$ or eg $p = \log_5\sqrt{125}$
	$p\log5 = \frac{3}{2}\log5 \Rightarrow p = \frac{3}{2}$	(A1)		Correct value of p must be explicitly stated
(ii)	$5^{2x} = \sqrt{125} = 5^p \Rightarrow x = 0.5p = 0.75$	B1F	1	Must be 0.5 × c's value of p SC: $x = 0.75$ with c's ans (a)(i) $5^{1.5}$ scores B1F
(b)	$3^{2x-1} = 0.05$ $(2x-1)\log3 = \log0.05$	M1		Take logs of both sides and use 3rd law of logs. PI eg by $2x-1 = \log_3 0.05$ seen
	$x = \dfrac{\log_{10}0.05}{2\log_{10}3} + \dfrac{1}{2}$	m1		Correct rearrangement to $x = \ldots$. PI
	$= -0.8634(165\ldots) = -0.8634$ to 4dp	A1	3	Condone > 4dp. Must see logs clearly **used** in solution, so NMS scores 0/3
(c)	$\log_a x = 2(\log_a 3 + \log_a 2) - 1$			
	$= 2\log_a(3\times2) - 1$	M1		A valid law of logs used
	$= \log_a(6^2) - 1$	M1		Another valid law of logs used
	$= \log_a 36 - \log_a a$	B1		$\log_a a = 1$ quoted or used or $\log_a \frac{x}{k} = -1 \Rightarrow \frac{x}{k} = a^{-1}$ OE
	$\log_a x = \log_a\left(\frac{36}{a}\right) \Rightarrow x = \frac{36}{a}$	A1	4	CSO Must be $x = \frac{36}{a}$ or $x = 36a^{-1}$
	Total		**10**	
	TOTAL		**75**	